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INVESTIGATION OF LASER PROPAGATION PHENOMENA

The Ohio State University
ElectroScience Laboratory
Department of Electrical Engineering
Columbus, Ohio 43212

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INVESTIGATION OF LASER PROPAGATION PHENOMENA

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PUBLICATION REVIEW

This technical report has been reviewed and is approved.

Raymond P. Urtz Jr.

RADC Project Engineer

ABSTRACT

This report deals with theoretical investigations in the area of linear atmospheric propagation phenomena and microturbulence statistics. It specifically deals with the examination of proper averaging times required for propagation experiments and with theoretical backup for phase structure function measurements. Finally, a bibliography on optical propagation which was prepared earlier has been updated.

CONTENTS

INTERIM TECHNICAL REPORT

INTRODUCTION

This is the first interim report under Contract No. F 30602-71-0132 entitled "Investigation of Laser Propagation Phenomena." This effort is aimed at providing theoretical support to the RADC Laser Propagation Program. The report covers the period January 1, 1971 to July 1, 1971.

The theoretical support is in the area of linear atmospheric propagation phenomena and microturbulence statistics. Other areas include theoretical support to the performance of propagation experiments and in the interpretation and processing of the data to ensure proper match between theory and experiment.

During the past six months effort has been concentrated in two areas; examination of proper averaging times required for propagation experiments and theoretical backup for phase structure function measurements. A bibliography on optical propagation prepared previously was also updated. The work on averaging times is in direct support to the experimental program at RADC because it is intended to supply information to be used in the data taking procedure. Some suggestions have already been incorporated into the data-taking procedure. The phase structure function work consists of the calculation of curves of interest in experiment design and interpretation. Specifically the curves predict the contribution of various portions of the path to the final measured value of the phase structure function for a spherical wave and for horizontal and slant paths in the turbulent boundary layer. These will be useful in interpretation of results and in future design of experiments. The bibliography is an extension of a previous work.

These items will now be considered in detail starting with the work on averaging times!

AVERAGING TIMES

In any type of measurement involving random quantities, the quantities of interest must be average values because it is only the average values which have any chance of being reproducible and predictable. Such quantities as means, variances and covariances, which are of interest in one application or another are all examples. However, in order for these quantities to be reproducible, and therefore meaningful, the averages must be taken over a representative set of values, a set large enough to include all the values to be encountered in the proper proportion. It is to answer the question of how much data to take in order to provide such a set of values that this study has been undertaken. This study is by no means complete. The chief result to date has been an examination of the literature to determine the various approaches used there. Results of this examination will be presented here.

There are several criteria for determining what a proper averaging time might be. They are based on precision desired and on the duplication of conditions required by assumptions in theoretical developments. For example, in theories of light beam propagation in random media the assumption of homogeneity and isotropy are almost always invoked. Further one may well use the central limit theorem to simplify predictions involving not too long propagation paths. In taking the data one generally uses the ergodic theorem and measures time averages rather than ensemble averages. Finally the upper and lower spectral limits of the desired data must be considered. All of these points depend on the averaging time for satisfactory physical implementation. In this section we present a review of the various approaches in the literature to questions involving averaging times, indicating assumptions and making a few limited observations. Much of the work described comes from the micrometeorological literature, the rest stemming from communication sources. The various authors consider various aspects of averaging times, based on the criterion selected, Ergodic principle, etc.; and on the type of data, analog vs sampled data, time signals or power spectra, etc. These approaches will now be reviewed.

<u>In the first approach</u> (Lumley and Panofsky, 1964; Davenport and Root, 1950; Bendat and Piersol, 1966) to questions of averaging times the question posed is how long must a time average be in order for it to represent an ensemble average. The discussion considers a random stationary analog signal, call it f(t). f(t) could be a direct signal or some function of a signal. The object is to relate the time average,

$$(1) \quad \overline{f(t)} = \frac{1}{T} \int_0^T f(t+t')\, dt'$$

and the ensemble average $\langle f(t) \rangle$. Ensemble averages are denoted by angular brackets. The measure of the difference between the two is taken to be the ensemble average square difference, $\sigma^2(t)$ between the two.

$$(2) \quad \sigma^2(t) = \langle [\frac{1}{T} \int_0^T f(t+t')\, dt' - \langle f(t) \rangle]^2 \rangle$$

$\sigma^2(t)$ is the ensemble variance of the time average.

An expression for an appropriate averaging time is worked out in terms of B(t'), the ensemble covariance of f(t), where

$$(3) \quad B(t') = \langle [f(t+t') - \langle f(t) \rangle]\, [f(t) - \langle f(t) \rangle] \rangle$$

and the autocorrelation $\rho(t) = B(t)/B(0)$. The basic expression is derived by first expressing Eq. (2) first as a double integral and then by putting it in terms of the autocorrelation to give

$$(4a) \quad \sigma^2 = \left\langle \frac{1}{T^2} \int_0^T \int_0^T [f(t+t') - \langle f(t) \rangle] [f(t+t'') - \langle f(t) \rangle] \, dt' \, dt'' \right\rangle$$

$$(4b) \quad = \frac{B(0)}{T^2} \int_0^T \int_0^T \rho(t'-t'') \, dt' \, dt''$$

The basic expression is simplified by transformation to sum and difference coordinates with the subsequent performance of one of the integrals.

$$(5) \quad t_1 = \frac{t'-t''}{1}, \quad t_2 = \frac{t'+t''}{2}$$

$$(6a) \quad \sigma^2 = \frac{2\,B(0)}{T^2} \int_0^T dt_1 \, \rho(t_1) \int_{\frac{1}{2}t_1}^{T-\frac{1}{2}t_1} dt_2$$

$$(6b) \quad = \frac{2\,B(0)}{T} \int_0^T \left(1 - \frac{t_1}{T}\right) \rho(t_1) \, dt_1$$

Equation (6b) can be further simplified if it is the case that the autocorrelation $\rho(t_1)$ drops to zero in a time much less than T. In that case the second term in the parenthesis inside the integral in Eq. (6b) is negligible. The result expressed in terms of I, the integral scale of the autocorrelation,

$$(7) \quad I = \int_0^\infty \rho(t_1) \, dt_1$$

is

(8) $T = \frac{2\,B(0)}{\sigma^2}\, I$

Equation (8) is put in a more useful form by expressing σ as a fraction, ε, of the ensemble mean, $\varepsilon = \sigma/\langle f \rangle$, giving

(9) $T = \frac{2\,B(0)}{\langle f \rangle^2}\, \frac{I}{\varepsilon^2}$

Equation (9) is the basic result. It is used to give averaging times formulas for several random quantities (Bendat and Piersol, 1966).

There are several points worth indicating about this result. The first is that it expresses an averaging time for f(t) in terms of the ensemble averages B(0), $\langle f(t) \rangle$ and an integral of an ensemble average! Thus, in order to estimate the averaging time required for time average to duplicate the ensemble average, it is necessary to already know some of these ensemble averages. To circumvent this problem, one might attempt a trial run, assume that the time averaged variance, mean, and integral scales are approximately realistic and estimate the averaging time. If the estimated value turns out to be what has already been used, then the measurements are at least consistent within the given framework.

A second point is indeed more significant: Eq. (9) says that for very long averaging times, the time and ensemble averages become identical! Thus, in situations where it is impossible or prohibitive to set up an ensemble of representative situations, then a time average can in principle be made to suffice. It was assumed that f(t) was stationary, so that there are not any long term trends to worry about!

The final point concerns an interpretation of Eq. (9). One might regard a long time average as a sort of ensemble average if the time record could be partitioned into successive segments each uncorrelated. Then each segment could be regarded as an independent ensemble member and the time and ensemble averages would be comparable. In that sense the integral scale of the autocorrelation gives a time proportional to a decorrelation time. The quantity

$$2I\,\frac{B(0)}{\langle f \rangle^2}$$

might be a little more representative of a decorrelation time. Then the decorrelation time divided by the square of the desired precision gives the requisite averaging time.

Alternatively, (Bendat and Piersol, 1966) one can state the same concept in different terms by defining the equivalent bandwidth, W, where

$$W = 1/4I.$$

Then the time-bandwidth product, WT is defined as the number of degrees of freedom and identified with the number of independent ensemble members. Thus the fractional deviation from ensemble average, ε, is inversely proportional to the time-bandwidth product, as might be intuitively expected.

If one assumes a gaussian distribution of data, the equation

$$T = \frac{2\, B(0)\, I}{\langle f\rangle^2\, \varepsilon^2}$$

can be further reduced. If the quantity to be measured is the mean square value of u, a gaussian random variable, then $B(0) = \langle (u^2 - \langle u^2\rangle)^2\rangle = 2\langle u^2\rangle^2$, $\langle f\rangle^2 = \langle u^2\rangle^2$, and

$$T = \frac{4\, I}{\varepsilon^2}$$

(Lumley and Panofsky, 1964). This equation, which assumes a Gaussian distribution, was applied to some microthermal data taken on March 23, 1971 at the Rome Air Development Center. Figure 1 shows the autocorrelation vs delay time for a single temperature sensor. The autocorrelation does approach a zero value so we assume that the integral scale does exist and trends in data are unobservable. From this graph we find that I = 0.1 sec. and

$$T = \frac{(4)(.1\ \text{sec})}{\varepsilon^2}$$

for	ε = 10%	T = 40 sec
	ε = 5%	T = 160 secs
	ε = 2%	T = 1000 secs

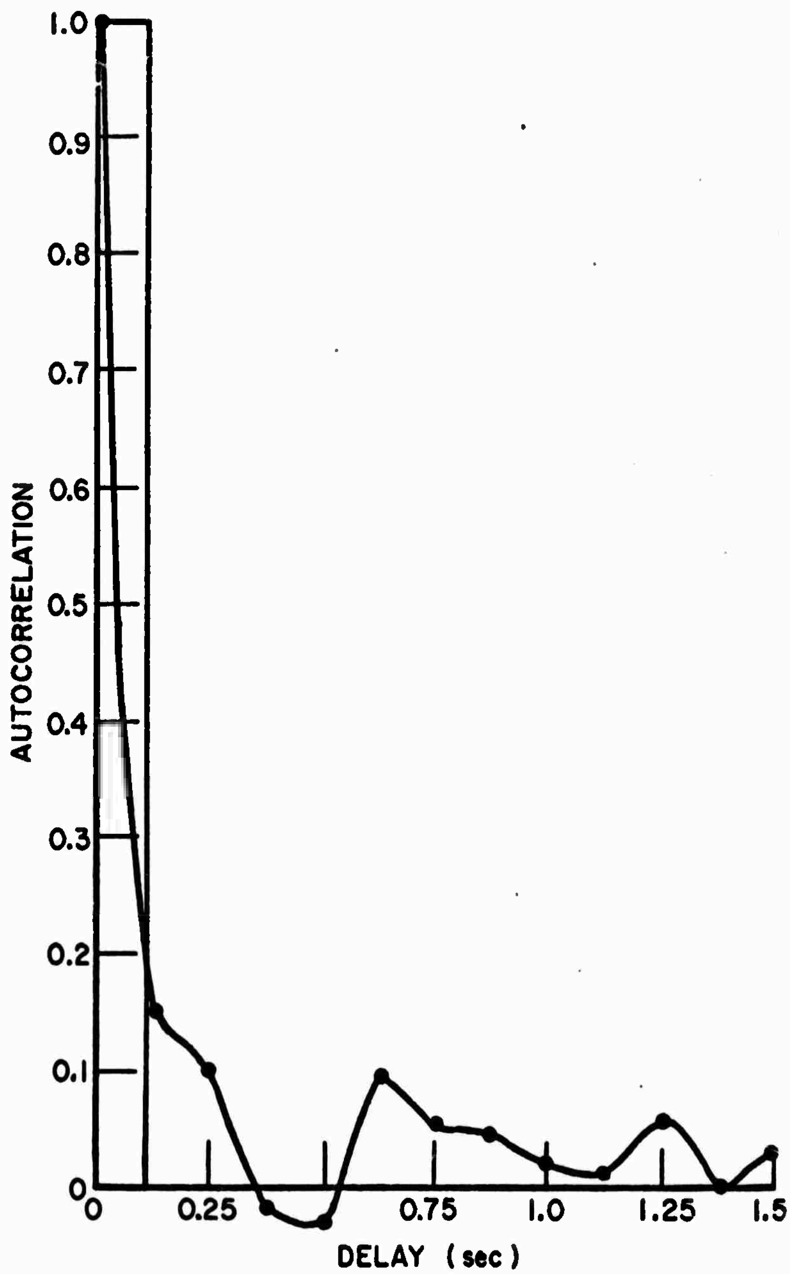


Fig. 1. Autocorrelation versus delay time of microthermal data.

A second approach (Charnock and Robinson, 1957) to the question of averaging times considers sampled data, again asking how long a data-taking duration is required. This method considers a random function of time sampled N times over a long interval T_o from which are calculated the mean

$$\bar{f} = \frac{1}{N} \sum_{i=1}^{N} f_i$$

and variance

$$B_N(0) = \frac{1}{N} \sum_{i=1}^{N} (f_i - \bar{f})^2$$

The subscript N denotes explicitely the number of data samples used. The N data samples are divided into sections of s samples each and the question asked is how large must s be in order for the section variances, averaged over the sections, to approximate the complete variance $B_N(0)$.

The approach is to consider the variances of the individual sections. The quantity used for comparison is $\overline{B}_s(0)$, the section variances averaged over all the sections

$$(12) \quad \overline{B}_s(0) = \frac{1}{m} \sum_{a=0}^{m-1} \frac{1}{s} \sum_{i=as+1}^{as+s} (f_i - \frac{1}{s} \sum_{as+1}^{as+s} f_j)^2$$

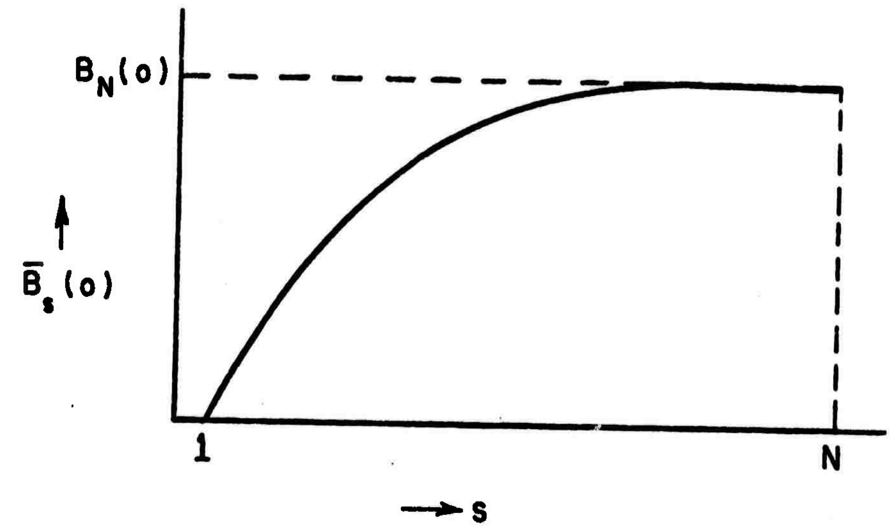
In Eq. (12) there are m sections of s=N/m samples. The subscript "a" indexes the individual sections. If the whole trace is considered as just one section then s = N, m = 1 and a = 0. In that case $B_N(0)$ is merely the complete sample variance indicated in Eq. (11). If there is only one sample in each section, then

$$s = 1,\ m = N \text{ and } \sum_{as+1}^{as+s} f_j = f_{a+1}$$

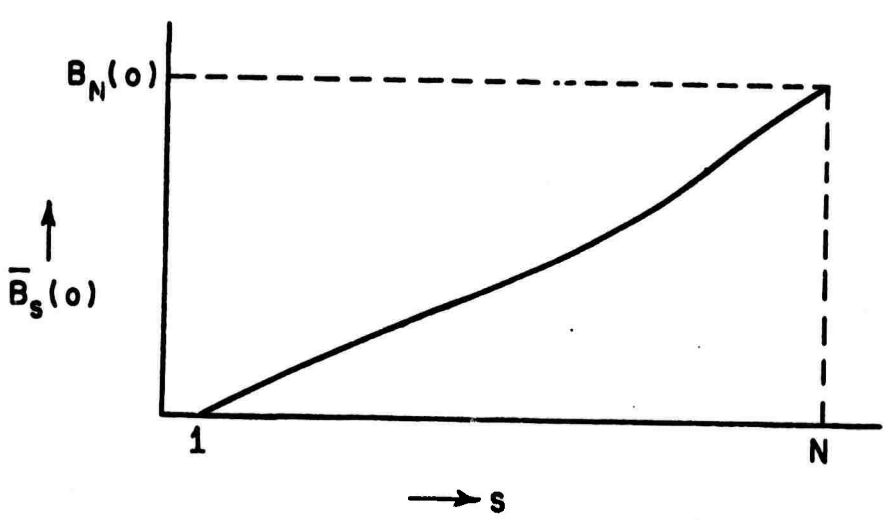
so that each term in the parenthesis in Eq. (12) is identically zero. Thus $\overline{B}_1(0) = 0$.

The question posed is what is the shape of the $\overline{B}_s(0)$ curve as a function of s? The behavior one would like to see is to have the mean of the section variances, $\overline{B}_s(0)$, approach $B_N(0)$ for s << N. Then the

total number of samples, N, and the averaging time, T_∞, is well above that required in practice. The minimum number of samples would be that for which $\overline{B}(0)$ initially comes within some preset region near $\overline{B}_N(0)$. For this situation the $\overline{B}_s(0)$ vs s curve would be similar to that of Fig. 2a. If the total number of samples is too small then the $\overline{B}_s(0)$ curve might have a shape more nearly like that of Fig. 2b. Thus it is



(a)



(b)

Fig. 2. Variance versus averaging time.

the shape of the $\overline{B}_S(0)$ curve that determines the appropriateness of the averaging time. The values of N and s must be sufficiently large with s << N to give the proper shape. In a sense this is also a self-consistency method as described because there is no a priori way stated for determining optimum values of s or T.

Continuing with this approach (Charnok and Robinson, 1957; Pasquill, 1962; Kahn, 1957 and Smith, 1962) the main points are made after Eq. (12) is simplified. To do this, add and subtract the long term average, $\overline{f}$, inside the parenthesis in Eq. (12), multiply out the square and simplify.

$$(13a)\quad \overline{B}_s(0) = \frac{1}{m}\sum_{a=0}^{m-1}\frac{1}{s}\sum_{as+1}^{as+s}\left(f_i-\overline{f}-\frac{1}{s}\sum_{as+1}^{as+s}(f_i-\overline{f})\right)^2$$

$$(13b)\quad = \frac{1}{m}\sum_{a=0}^{m-1}\frac{1}{s}\sum_{as+1}^{as+s}\left[(f_i-\overline{f})^2 - 2(f_i-\overline{f})\frac{1}{s}\sum_{as+1}^{as+s}(f_i-\overline{f}) + \left(\frac{1}{s}\sum_{as+1}^{as+s}(f_i-\overline{f})\right)^2\right]$$

$$(13c)\quad = \frac{1}{m}\sum_{a=0}^{m-1}\left[\frac{1}{s}\sum_{as+1}^{as+s}(f_i-\overline{f})^2 - \left(\frac{1}{s}\sum_{as+1}^{as+s}(f_i-\overline{f})\right)^2\right]$$

Equation (13c) was the starting point for Charnok and Robinson. In the first term in Eq. (13b) the double summation is merely the complete sum over all samples so this term is merely $B_N(0)$. Thus Eq. (12) reduces to

$$(14)\quad \overline{B}_s(0) = B_N(0) - \frac{1}{m}\sum_{a=0}^{m-1}\left(\frac{1}{s}\sum_{as+1}^{as+s}(f_i-\overline{f})\right)^2$$

The second term in Eq. (14) contains the difference between the section means and the long term mean and indeed should go to zero if the sections become sufficiently long, leaving $B_N(0)$ as desired.

The second term in Eq. (14) can be further simplified by writing it in terms of the covariance $B_0(k)$, where

$$(15)\quad B_0(k) = \frac{1}{m(s-k)}\sum_{a=0}^{m-1}\sum_{i=as+1}^{as+s-k}(f_i-\overline{f})(f_{i+k}-\overline{f})$$

Thus multiplying out the terms in the double summation in the second term in Eq. (14) and regrouping using Eq. (15) gives

$$\text{(16a)}\quad \overline{B}_s(0) - B_N(0) = \frac{1}{s}\left[\frac{1}{s}B_o(1-s) + \frac{2}{s}B_o(2-s)+\dots+\frac{s}{s}B_o(0)+\dots+\frac{1}{s}B_o(s-1)\right]$$

$$\text{(16b)}\quad = \frac{1}{s}\sum_{k=-(s-1)}^{(s-1)} \left(1 - \frac{|k|}{s}\right) B_o(k).$$

If s and N are sufficiently large, then the summation in Eq. (16b) can be approximated by an integral

$$\text{(17a)}\quad \overline{B}_s(0) - B_N(0) \doteq \frac{1}{s}\int_{-s}^{s} \left(1 - \frac{|k|}{s}\right) B(k)\,dk$$

$$\text{(17b)}\quad = \frac{2\overline{B}(0)}{s}\int_{0}^{s} \left(1 - \frac{|k|}{s}\right) \frac{B(k)}{B(0)}\,dk$$

assuming B(k) is a symmetric function. The right hand side of Eq. (17b) then has a form identical with Eq. (6b) if we put

$$\text{(17c)}\quad \frac{T_o k}{N} = t_1 \quad\text{and}\quad \frac{T_o s}{N} = T.$$

T_o is as before time required for all N samples. The result is

$$\text{(17d)}\quad \overline{B}_T(0) = B_N(0) - \frac{2\,B_N(0)}{T}\int_{0}^{T} \left(1 - \frac{t_1}{T}\right) \frac{B(t_1)}{B_N(0)}\,dt_1$$

Equation (17d) can be derived in a slightly different fashion, and a more complete comparison between the approaches of Lumley and Panofsky presented first and of Charnok and Robinson can be presented by interpreting the sum in Charnok and Robinson's approach in a particular way and by assuming a continuous rather than a sampled

trace. Thus consider the trace of duration T_0 to be divided up into sections each of duration T. Then identify the m intervals with members of an ensemble. One then computes the time average and the time variance, i.e. the time average square deviation, about the time average for each ensemble member. The question asked then is how does this ensemble average of the time variances depend on averaging time, T, e.g., how long must T be in order for the ensemble average time variance to approach the ensemble variance.

With the interpretations of the sums in mind we can rewrite Eq. (12) by replacing the sum over "a" by an ensemble average and replacing the sum over "i" by a time average. Equation (12) becomes

$$(18a)\quad B_T(0) = \left\langle \frac{1}{T}\int_0^T \{f(t) - \frac{1}{T}\int_0^T f(t)\,dt\}^2\,dt \right\rangle$$

and Eq. (14) similarly becomes

$$(18b)\quad B_T(0) = B(0) - \left\langle \left[\frac{1}{T}\int_0^T f(t)\,dt\right]^2 \right\rangle$$

The second term on the right hand side of Eq. (18b) is identical to the expression in Eq. (4a) and can be similarly simplified giving

$$(18c)\quad B_T(0) = B(0) - \frac{2\,B(0)}{T}\int_0^T (1 - \frac{t_1}{T})\,\rho(t_1)\,dt_1$$

which is the same as Eq. (17d), if we identify $B_N(0)$ with the ensemble variance. This reinforces the concept of individual time samples being independent and being regarded as individual ensemble members.

It is interesting to note here that the same integral is used in two different situations. In Eq. (6b) it gives the mean square ensemble average difference between time and ensemble averages, denoting the approach of these two averages with increased averaging time. In Eq. (18c) it gives the ensemble average of the time variance, again denoting the approach of the two with increasing averaging time.

There is one other result of Charnock and Robinson; a procedure for processing the function $B_S(0)$ in Eq. (17b) and Fig. 2 to give directly the covariance B(s). This expression is

$$(19a)\quad B(s) = B(0) - \frac{1}{2}\frac{\partial^2}{\partial s^2}\left(s^2 B_s(0)\right)$$

This result is obtained by differentiating $s^2B_s(0)$ with $B_s(0)$ taken from Eq. (17a) to give

$$(19b)\quad \frac{\partial^2}{\partial s}(s^2B_s(0)) = 2sB_N(0) - \int_{-s}^{s} B(k)\,dk$$

$$(19c)\quad \frac{\partial^2}{\partial s^2}(s^2B_s(0)) = 2B_N(0) - 2B(s)$$

Using the fact that

$$(20)\quad B_N(0) = \frac{1}{N}\sum_{i}^{N}(f_i-\overline{f})^2 \equiv B(0)$$

and rearranging gives Eq. (19a). Thus, from averaging times it is possible to pull out data on other than just averaging times.

It is also possible to express the approach of the averaging time to its proper value as given in Eqs. (16b) and (17a) using a spectral description, (Charnock and Robinson, 1957; Ogura, 1957; Pasquill, 1962 and Smith, 1962). Thus, for example, introduce the power spectrum $S(\omega)$ given by

$$(21)\quad B\left(\frac{Nt}{T}\right) = B(t) = \int_{-\infty}^{\infty} s(\omega)\, e^{j\omega t}\,d\omega$$

into Eq. (17a)

$$(22)\quad B_s(0) = B_N(0) - \frac{1}{T}\int_{-T}^{T} dt\,\left(1 - \frac{t}{T}\right)\int_{-\infty}^{\infty} s(\omega)\, e^{j\omega t}\,d\omega$$

Performing the integration and using Eqs. (20 and (21) gives

$$(23)\quad B_s(0) = \int_{-\infty}^{\infty} s(\omega)\left(1 - \frac{\sin^2(\frac{1}{2}\omega T)}{(\frac{1}{2}\omega T)^2}\right)d\omega$$

Equation (23) indicates the well-known fact that a finite averaging time acts like a frequency filter. The filter function in this case is $\{1 - [\sin(\frac{1}{2}\omega T)/(\frac{1}{2}\omega T)]^2\}$ which acts like a high-pass filter, passing frequencies higher than $f_o = 1/T$. Thus the period of recording should be sufficiently long to faithfully reproduce the lowest frequencies desired.

To summarize the results so far, it appears that an acceptable averaging time for approach of time average and variance to the corresponding ensemble averages might be deduced by plotting a set of data in a manner indicated in Fig. 2a and adjusting the total amount of data and length of sections until a curve similar to Fig. 2a is obtained. The time average will then be a good representation to the ensemble average. One should then check that the section durations are sufficiently long that the lowest frequencies desired are reproduced. Finally it is possible to obtain the covariance from the curve in Fig. 2a.

A third approach to averaging time considers measurement of power spectra (Blackman and Tukey, 1958) of random stationary analog data, normally distributed. Several steps of data processing are used. First the data are processed to give the covariance which is then multiplied by a bellshaped curve which forces the product to stay at zero after some lag time T_m after which the covariance has become very small. The product is then transformed to give the power spectrum and the power spectrum is then averaged over an ensemble of situations. The question posed is again, how long must the original data trace be in order for the power spectral value to be within a predetermined range, in dB, say ninety percent of the time.

The averaging time question is answered quite simply for power spectra because they follow chi-square statistics. For such a case the confidence limits are used to determine the number of degrees of freedom, k, which are in turn related to the power spectral mean and variance as shown in Eq. (25).

$$(25) \qquad k = \frac{2\langle P(\omega)\rangle^2}{(\langle P(\omega) - \langle P(\omega)\rangle)^2}$$

These spectral averages are in turn related to the averaging time T and the cutoff time T_m to give the desired results. The basic steps in the derivation of this method will be outlined. The reader is referred to the original source (Blackman and Tukey, 1958) for a more detailed discussion.

To continue with the derivation outline, then, start with the time trace X(t) of a single ensemble member. For that case the covariance is given by (neglecting end of trace effects)

$$B(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} X(t-\tfrac{\tau}{2})\, X(t+\tfrac{\tau}{2})\, dt \tag{26}$$

For this same time trace, the power spectral density would then be

$$P(\omega) = \int_{-\infty}^{\infty} B(\tau)\, D(\tau) \cos \quad d \tag{27a}$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} d\tau\, D(\tau) \cos \omega\tau \int_{-T/2}^{T/2} X(t-\tfrac{\tau}{2})\, X(t+\tfrac{\tau}{2})\, dt \quad . \tag{27b}$$

$D(\tau)$ is a bell shaped function with the restrictions

$$D(\tau) = 1 \qquad \tau = 0$$
$$D(\tau) = 0 \qquad \tau \geq T_m$$

where T_m is the value of τ at which the covariance becomes neglegibly small. $D(\tau)$ eliminates noise arising from fluctuations for $\tau > T_m$. The ensemble average of the power spectrum would then be

$$\langle P(\omega)\rangle = \frac{1}{T} \int_{-\infty}^{\infty} d\tau\, D(\tau) \cos \omega\tau \int_{-T/2}^{T/2} \langle X(t-\tfrac{\tau}{2})\, X(t+\tfrac{\tau}{2})\rangle dt \tag{28a}$$

$$= \int_{-\infty}^{\infty} d\tau\, D(\tau)\, B_o(\tau) \cos \omega\tau \tag{28b}$$

where

$$B_o(\tau) = \langle X(t+\tfrac{\tau}{2})\, X(t-\tfrac{\tau}{2})\rangle \tag{29}$$

is the ensemble covariance of $X(t)$.

The mean square of the spectral density can also be estimated Starting with Eq. (27b), we have for a single trace,

$$(30) \quad P^2(\omega) = \frac{1}{T^2}\int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' \; D(\tau)\, D(\tau')\, X(t-\tfrac{\tau}{2})$$

$$X(t+\tfrac{\tau}{2})\, X(t'-\tfrac{\tau'}{2})\, X(t'+\tfrac{\tau'}{2}) \cos \omega\tau \cos \omega\tau'$$

Taking the ensemble average of both sides of Eq. (30) would give a fourfold average under the integral sign. This can be evaluated assuming the data, (the X's) are normally distributed. The result is

$$(31) \quad \langle X(t+\tfrac{\tau}{2})\, X(t-\tfrac{\tau}{2})\, X(t'+\tfrac{\tau'}{2})\, X(t'-\tfrac{\tau'}{2})\rangle = B_o(t-t'+\tfrac{\tau+\tau'}{2})\, B_o(t-t'-\tfrac{\tau+\tau'}{2})$$

$$+ B_o(t-t'+ \tfrac{\tau-\tau'}{2})\, B_o(t-t'- \tfrac{\tau-\tau'}{2}) + B_o(\tau)\, B_o(\tau')$$

Equations (30) and (31) are evaluated in the spectral domain. The result, when combined with Eq. (28a), is

$$(32) \quad \langle (P(\omega)-\langle P(\omega)\rangle)^2\rangle = \frac{1}{2\pi T}\int_0^{\infty} H^2(\omega,\omega_1)\, P^2(\omega_1)\, d\omega_1$$

where $H(\omega,\omega_1)$ is a symmetrized transform of $D(\tau)$. Finally Eqs. (29) and (32) are evaluated assuming the $P(\omega)$ does not change appreciably over a spectral range $2\pi/T_m$. The results, when inserted into Eq. (25), are

$$(33) \quad k = \frac{2\langle P(\omega)\rangle^2}{\langle (P(\omega)-\langle P(\omega)\rangle)^2\rangle} = \frac{2T\left[\int_0^{\infty} P(\omega)H(\omega)d\omega\right]^2}{2\pi \int [P(\omega)H(\omega)]^2 d\omega} = \frac{2T}{T_m}$$

Equation (33) is the basic result. It says that the number of degrees of freedom as determined from the chi-square distribution and the desired precision is given by $2T/T_m$. This is then interpreted, as before, as the number of statistically independent sections of data.

As another approach we might indicate what one could also apply the chi-square distribution to finding the proper averaging time when dealing specifically with the variance of a normally distributed stationary

random variable. For example, let a time trace be divided into sections each T_m long, so that they are essentially uncorrelated. Then the mean variance formed from the variances of the individual traces should follow a chi-square distribution. Thus, one need merely use the chi-square tables in conjunction with the desired precision to determine the requisite number of degrees of freedom. This then gives the number of sections of length T_m in the total data run.

To summarize, the study of averaging times must concern the criterion for setting the averaging time, and the exact quantity measured. Several examples appearing in the literature have been considered associated with the approach of the time average value of the mean and the variance to the ensemble value and the precision of the power spectrum and variance to the desired values.

PHASE STRUCTURE FUNCTION STUDIES

In the examination of phase structure function various curves have been plotted to provide useful information for comparing results and for planning experiments. There are two types of curves. The first type shows spherical wave phase structure function as a function of separation for various ranges normalized to refractive index parameter, C_N^2. This data is intended for direct comparison with experimental results. The second type of curve shows the contribution to spherical wave phase structure function of various portions of the propagation path. These curves of relative contribution versus distance are plotted for various separations, for various values of the parameter $2kL_0^2/L$ where $\lambda = 2\pi/k$ is the wavelength, L is the total range and L_0 is the turbulence outer scale. They are also normalized to C_N^2. Also included are various paths, including horizontal and slightly inclined paths with the beam propagation both upward and downward. These curves are intended to be useful in the design and interpretation of future experiments and in the possible interpretation of present data.

The curves are based on expressions already in the literature (Carlson, 1969) derived using the Rytov approximation. The basic expression for $D_s(\rho)$ is

$$(34)\qquad D_s(\rho) = 8\pi^2 k^2 \int_0^\infty [1-J_0(\kappa\rho)] \int_0^L \left(\frac{L}{Z}\cos\left(\frac{L(L-Z)\kappa^2}{2kZ}\right)\right)^2 \Phi_N\left(\frac{\kappa L}{Z}\right) dZ\, \kappa\, d\kappa$$

where
- $\lambda = 2\pi/k$ = light wavelength
- ρ = observation point separation
- κ = spatial frequency
- L = range
- Z = distance from transmitter along propagation path
- Φ_N = refractive index spatial spectrum.

The index spatial spectrum used is the Von Karman spectrum given in Eq. (35).

$$\text{(35a)} \quad \Phi_N(\kappa) = \frac{0.033\, C_N^2(H)\, e^{-(\kappa \ell_o/5.92)^2}}{(\kappa^2 + 1/L_o^2)^{11/6}}$$

$$\text{(35b)} \quad \doteq 0.033\, C_N^2(H)\, (\kappa^2 + L_o^{-2})^{-11/6}$$

where ℓ_o = turbulence inner scale, H = height.

The approximate form in Eq. (35b) was used because the phase structure function is not generally measured at the separations comparable with the inner scale, and the curves are for comparison with measured values. For the slant paths the structure parameter was taken to vary with height to the power (-4/3), (Wyngard et al, 1971) and the outer scale proportional to altitude. These are valid in the first 500 ft for unstable air and well developed turbulence.

$$\text{(36a)} \quad C_N^2(H) = C_N^2(H_o)(H/H_o)^{-4/3}$$

$$\text{(36b)} \quad L_o(H) = L_o(H_o)\left(\frac{H}{H_o}\right)$$

$$\text{(36c)} \quad \frac{H}{H_o} = 1 + \frac{Z}{L}\left(\frac{H_L - H_o}{H_o}\right)$$

where H_o = transmitter height
H_L = receiver height.

Equation (36c) written with distance along the path as a parameter was the actual expression used.

The calculations were performed on a digital computer using expressions derived from Eqs. (34) - (36). To find the incremental pathlength contributions to $D_S(\rho)$ the order of integrations in Eq. (34) was reversed, the resultant inner integral giving the desired incremental path contributions. The calculations were made using normalized dimensionless parameters

(37a) $u \equiv L_o(H_o)\kappa$

(37b) $\nu \equiv Z/L$

(37c) $y \equiv 2k\, L_o^2\, (H_o)/L$

(37d) $C_o \equiv C_N^2(H_o)\, k^{7/6} L^{11/6} = C_\ell^s(0)/0.13$

(Tatarski, 1961) giving to Eq. (34) the form

$$(38a)\quad D_s(\rho/L_o) = \{0.033 \times 8\pi^2 C_o \times (\tfrac{y}{2})^{5/6}\} \times \int_0^1 d\nu\, F(\nu)$$

where

$$(38b)\quad F(\nu) = \int_0^\infty du(1-J_o(\tfrac{\rho u}{L_o}))\, \frac{u}{\nu^2}\, \cos^2\left(\frac{u^2(\frac{1}{\nu}-1)}{y}\right) \times \left[\left(\frac{1}{1+\nu\left(\frac{H_L - H_o}{H_o}\right)}\right)^2 + \frac{u^2}{\nu^2}\right]^{-11/6}$$

$$\times\ (1 + \nu(\frac{H_L - H_o}{H_o}))^{-4/3}$$

Of the reduced parameters, u is a reduced spatial frequency, ν is reduced range, y is the Fresnel number for an aperture the size of the outer scale, and C_o is proportional to the spherical wave log amplitude variance, often used as a scaling parameter.

Figures 3a and 3b show the contribution to the normalized phase structure function for 8 separations (normalized) and for $y = 10^{+3}$. The normalized function, $F(\nu)$ is insensitive to the parameter y and hence Fig. 3a covers all of the experimentally interesting range of y. Fig. 3b shows that y has a larger (but still small) effect on $F(\nu)$ for the smaller separations (ρ/L_o). This means that the range dependence in $F(\nu)$ is very nearly independent of wavelength, the values $10^3 \le y = \le 10^5$ cover both 0.6μ and 10μ for ranges of interest. The wavelength dependence is in the $y^{5/6}$ part of the normalization constant in front of the integrals. From Figs. 3a and 3b it is evident that the most important contributions to D_s are made near the receiver for a horizontal path.

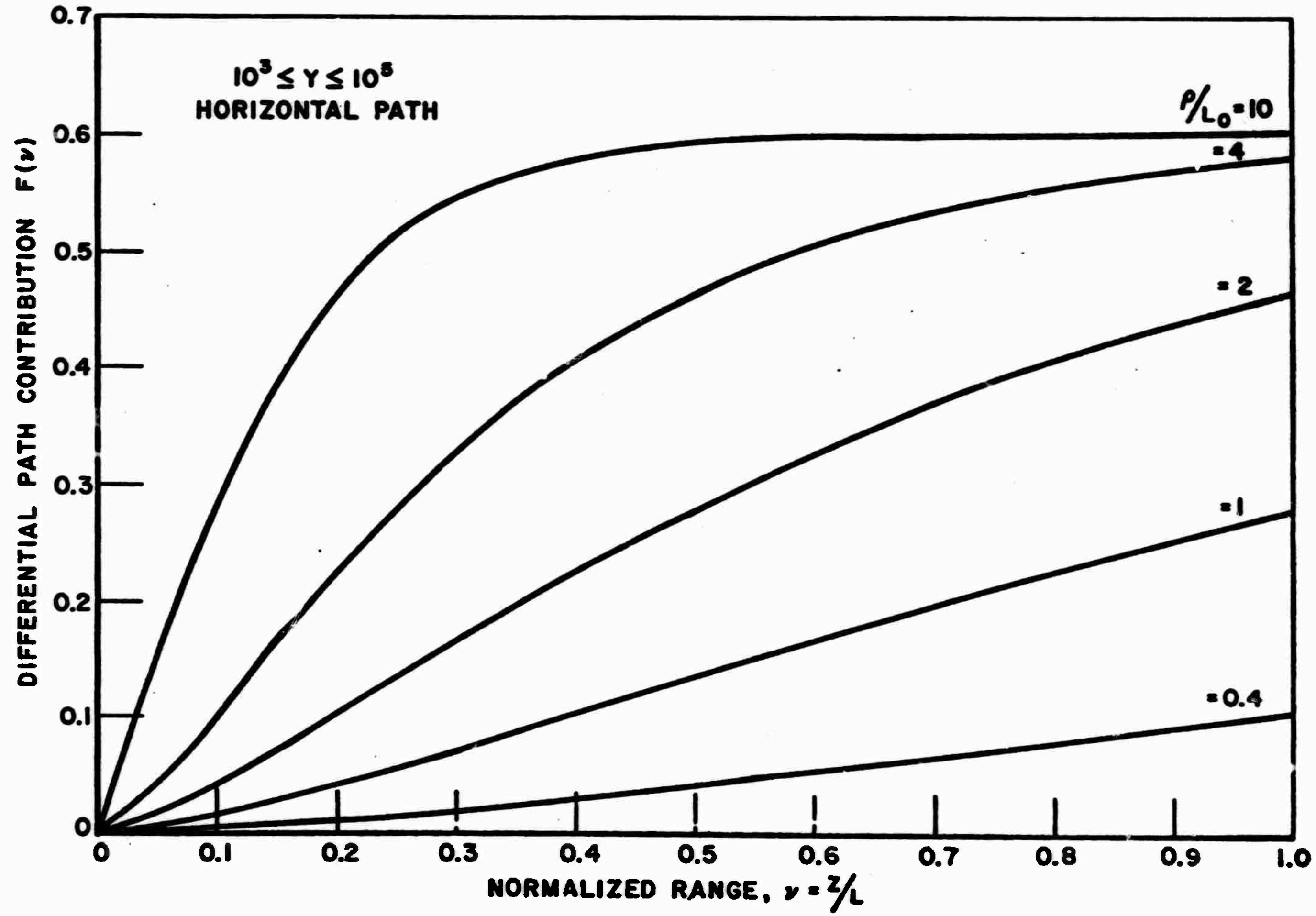


Fig. 3a. Differential spherical wave phase structure function contribution vs normalized range for five normalized separations.

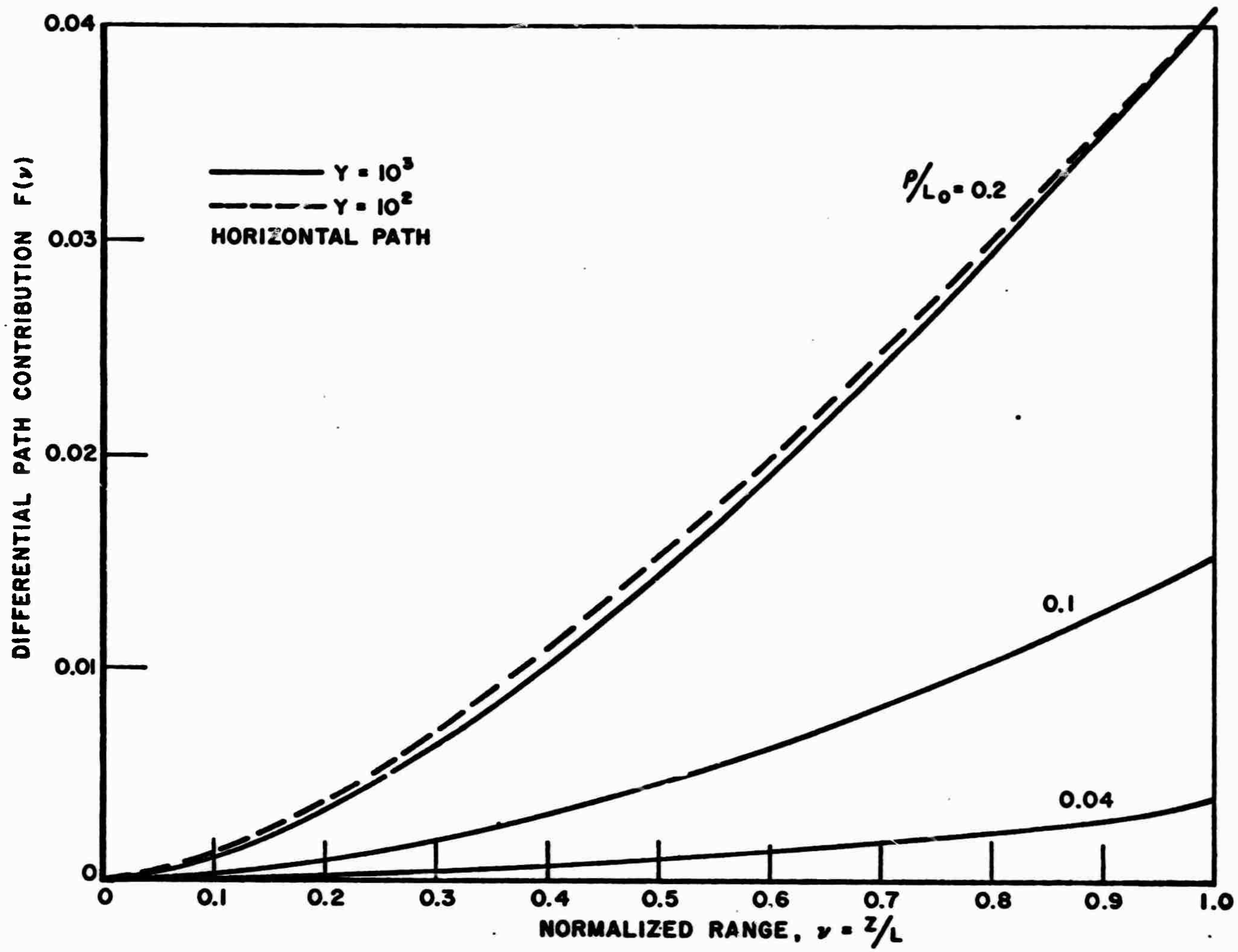


Fig. 3b. Differential spherical wave phase structure function contribution vs normalized range for three normalized separations.

Figures 4 and 5 show the normalized D_S integrand for an upward and downward slant path respectively. In the upward path the curve amplitude has "saturated", so that a more nearly constant contribution to range results. In the downward path the turbulence effects are strongly concentrated at the receiver.

Having the functions in Figs. 3a and 3b it is a simple matter to calculate the complete phase structure function.

The unnormalized phase structure function, for direct comparison with experimental results, is presented in Figs. 6a, 6b, and 6c for the three ranges of immediate interest.

Having developed the programs to calculate range dependence of the phase structure function, it was also possible with only minor modifications to calculate the range dependence of the small aperture angle of arrival correlation functions.

The small aperture elevation (B_α) and the azimuth (B_β) angle of arrival correlation functions are related to the phase structure function by

$$(39) \qquad B_\alpha(\rho) = \frac{1}{2k^2} \frac{\partial^2 D_s(\rho)}{\partial \rho^2}$$

$$(40) \qquad B_\beta(\rho) = \frac{1}{2k^2 \rho} \frac{\partial D_s(\rho)}{\partial \rho}$$

Equations 34, 39, and 40 may be used to find the contribution to B_α and B_β from various ranges, and (for simplicity) a horizontal path.

$$(41) \qquad B_\beta\left(\frac{\rho}{L_o}\right) = 4\pi^2 \times .033\; C_N^2(H_o) \frac{L}{L_o^{1/3}} \int_0^1 G(\nu)\, d\nu$$

$$G(\nu) = \int_0^\infty \frac{J_1\left(\frac{\rho u}{L_o}\right)}{\left(\frac{\rho u}{L_o}\right)} \frac{u^3}{\nu^2} \cos^2\left(\frac{u^2}{y}\left\{\frac{1}{\nu} - 1\right\}\right)\left(1 + \frac{u^2}{\nu^2}\right)^{-11/6} du$$

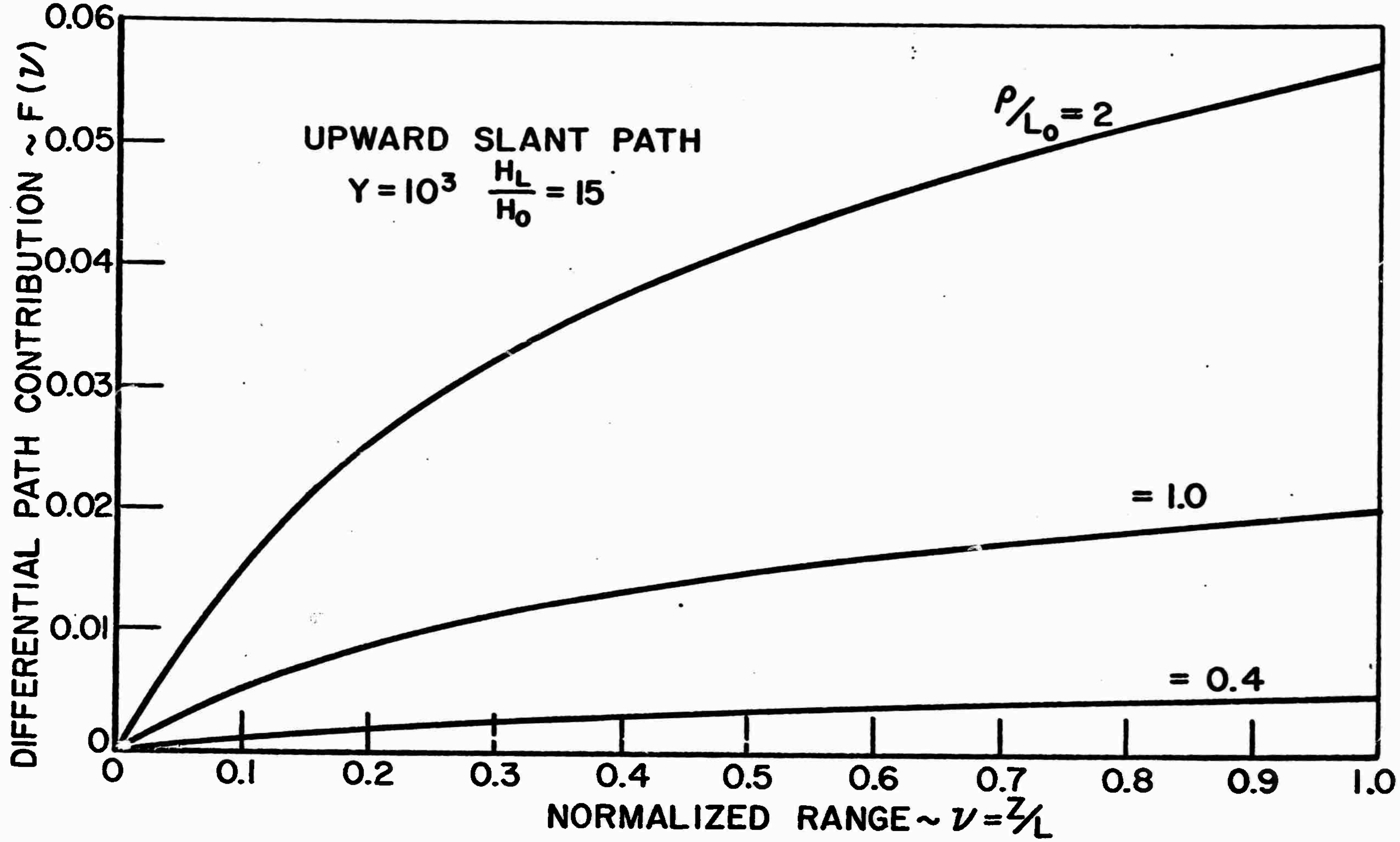


Fig. 4. Differential spherical wave phase structure function contribution vs normalized range for three normalized separations.

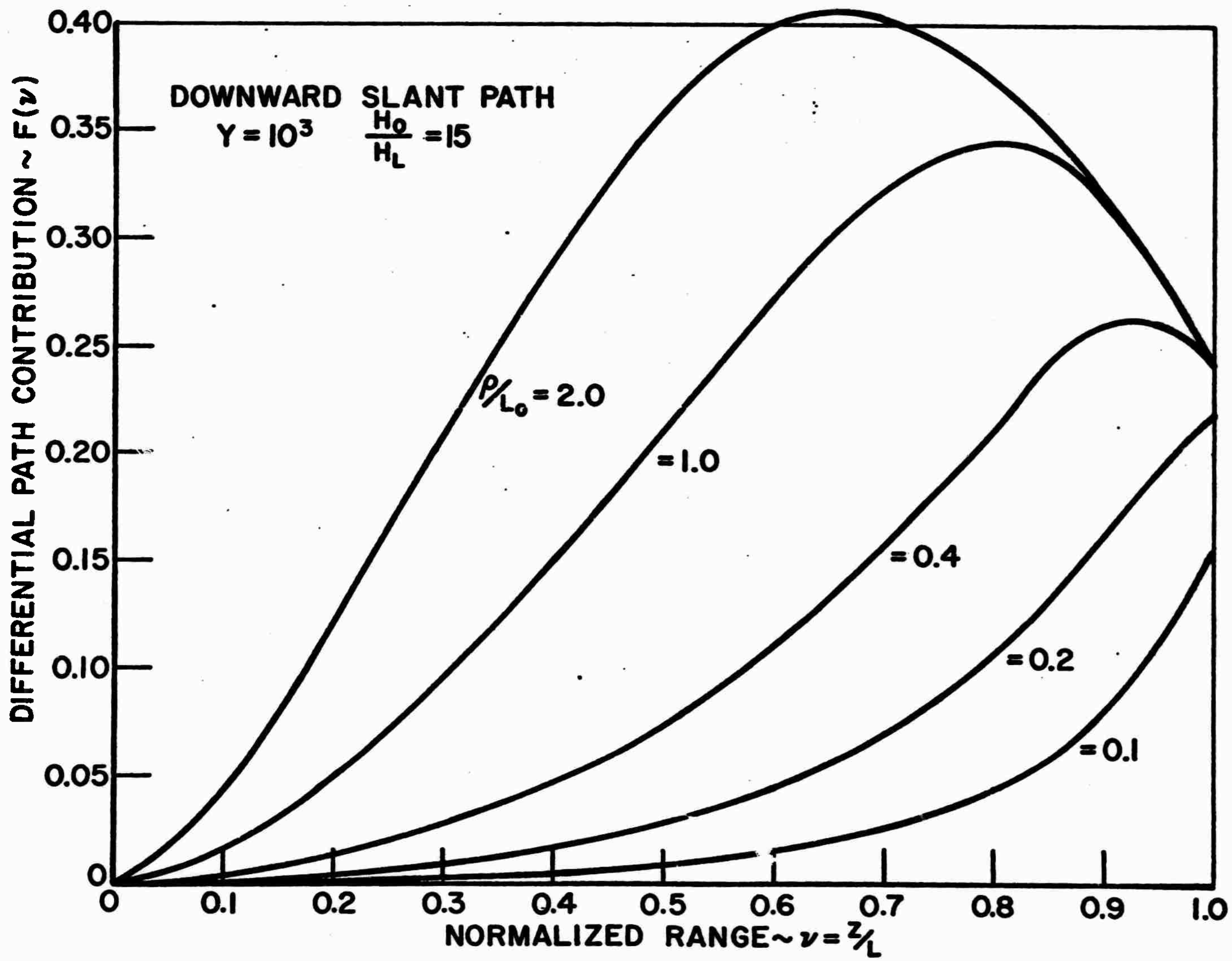


Fig. 5. Differential spherical wave phase structure function contribution vs normalized range for five normalized separations.

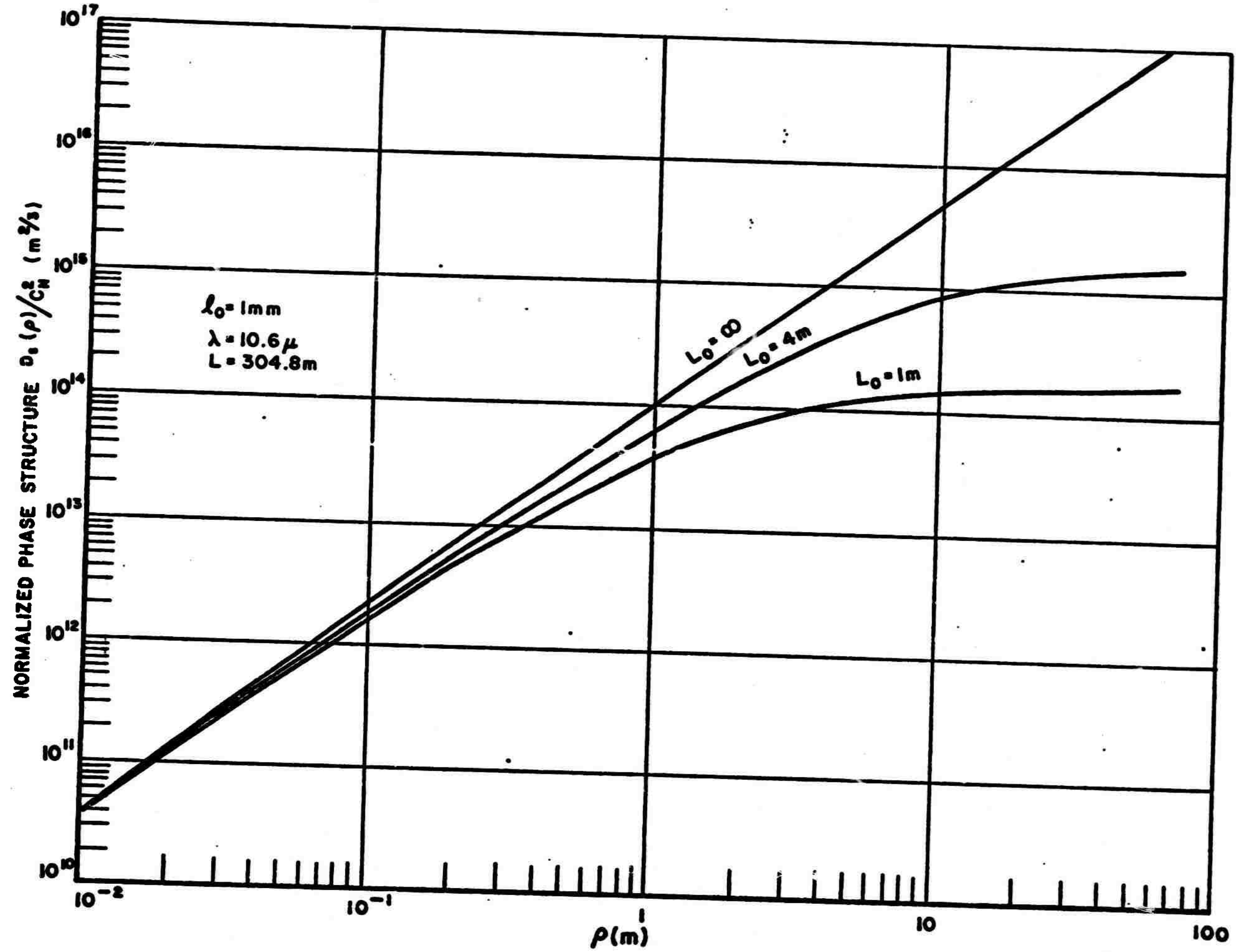
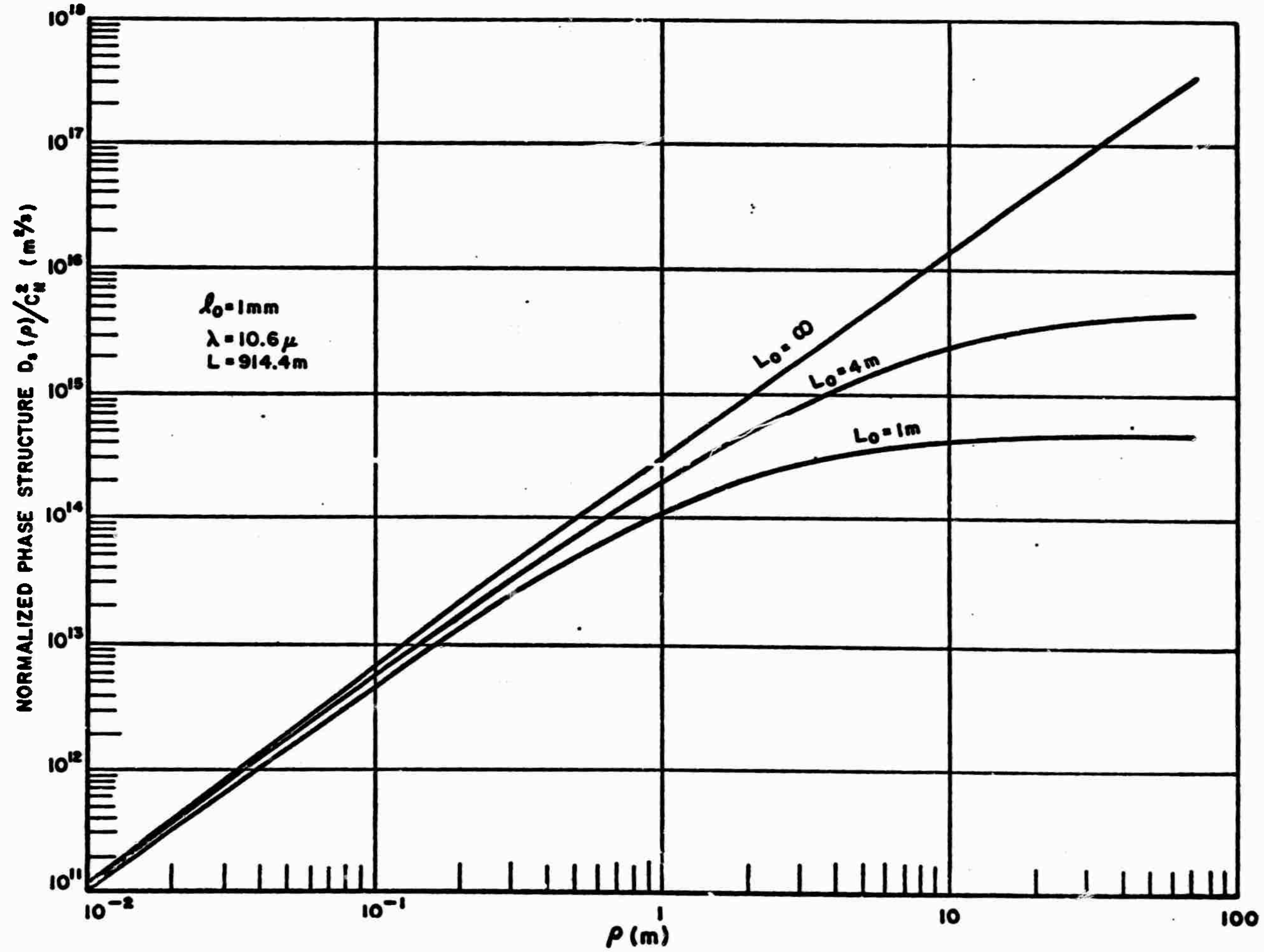


Fig. 6(a,b,c). Horizontal path spherical wave phase structure function vs separation and three outer scales.



NORMALIZED PHASE STRUCTURE $D_s(\rho)/C_N^2$ ($m^{5/3}$)
10^{18}
10^{17}
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
ℓ_0 = 1mm
λ = 10.6 μ
L = 914.4m
$L_0 = \infty$
L_0 = 4m
L_0 = 1m
10^{-2}
10^{-1}
1
10
100
ρ (m)

Fig. 6b. (Cont.)

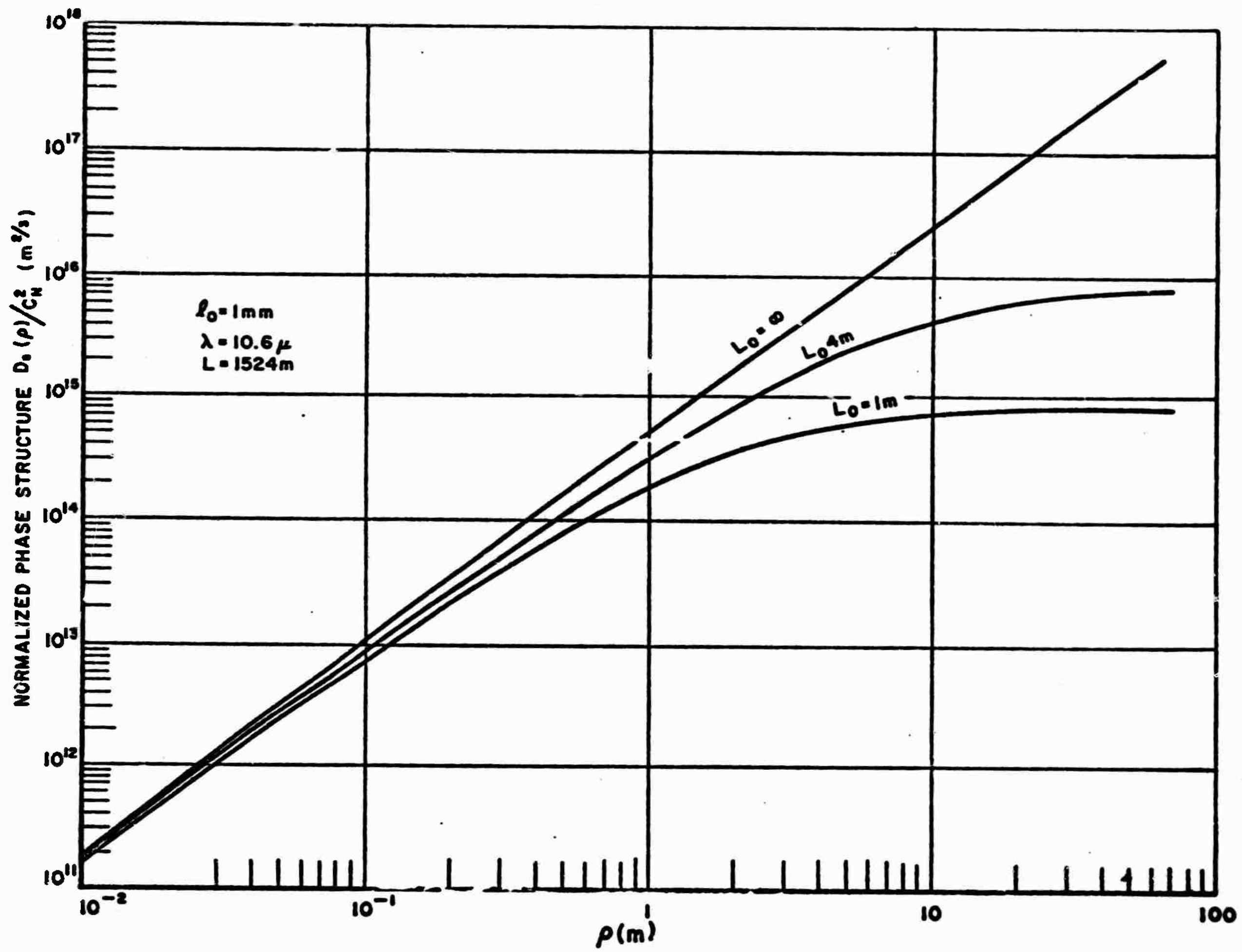

NORMALIZED PHASE STRUCTURE $D_s(\rho)/C_N^2$ (m^2/s)
ℓ_0 = 1mm
λ = 10.6 μ
L = 1524m
$L_0 = \infty$
L_0 4m
L_0 = 1m
ρ(m)

Fig. 6c. (Cont.)

$$(42) \quad B_\alpha\left(\frac{\rho}{L_o}\right) = 4\pi^2 \times .033\, C_N^2(H_o)\, \frac{L}{L_o^{1/3}} \int_0^1 H(\nu)\, d$$

$$H(\nu) = \int_0^\infty J_o\left(\frac{\rho u}{L_o}\right) \frac{u^3}{\nu^2} \cos^2\left(\frac{u^2}{y}\left\{\frac{1}{\nu} - 1\right\}\right)$$

$$\times \left(1 + \frac{u^2}{\nu^2}\right)^{-11/6} du - G(\nu).$$

Typical examples are shown in Figs. 7a and 7b.

The numerical calculations were done with double precision (17 significant figures) arithmetic. A 96 point Gaussian quadrature integration subroutine was used to do the inner integrations. The numerical integration limits are such as to yield better than 1.% accuracy.

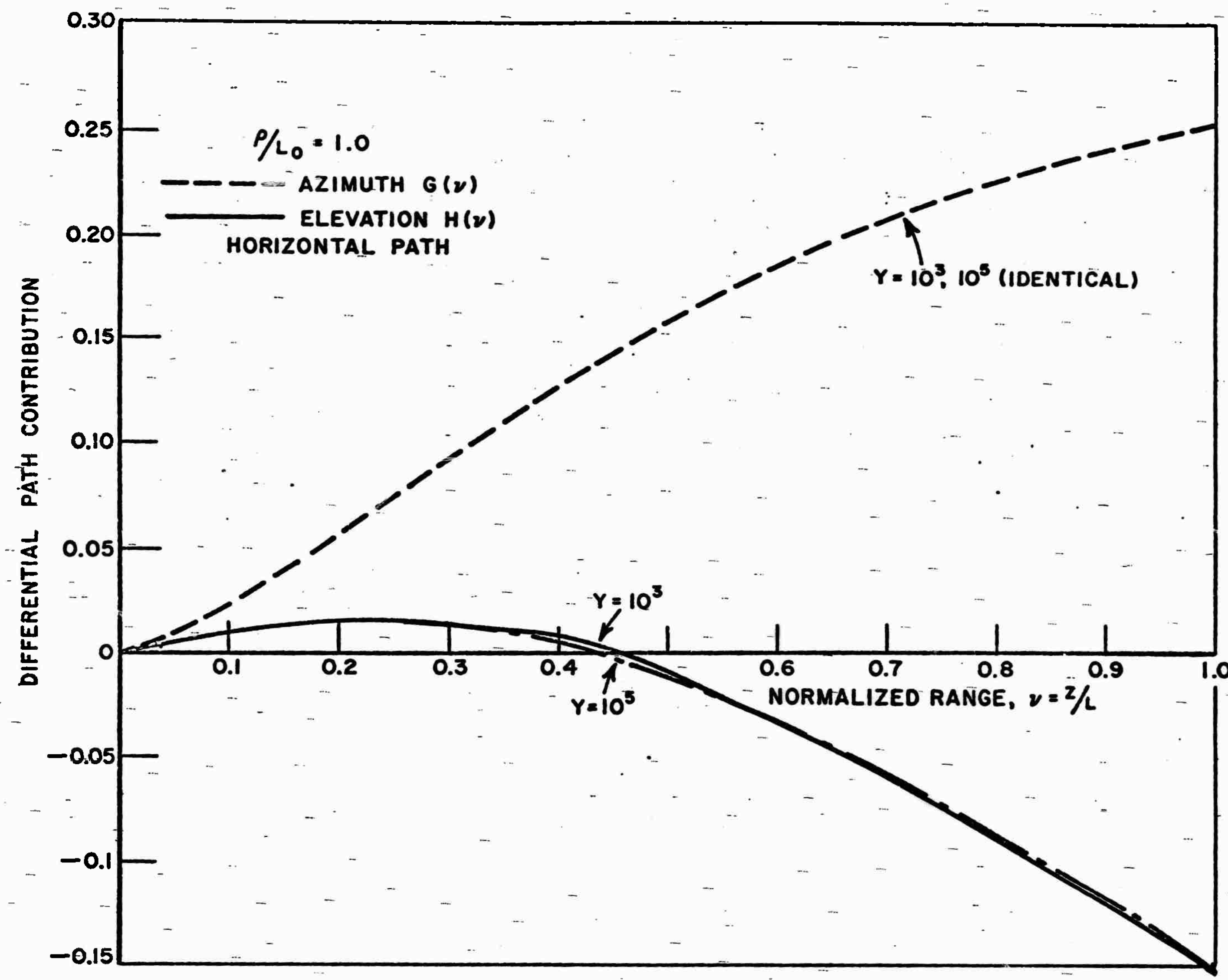
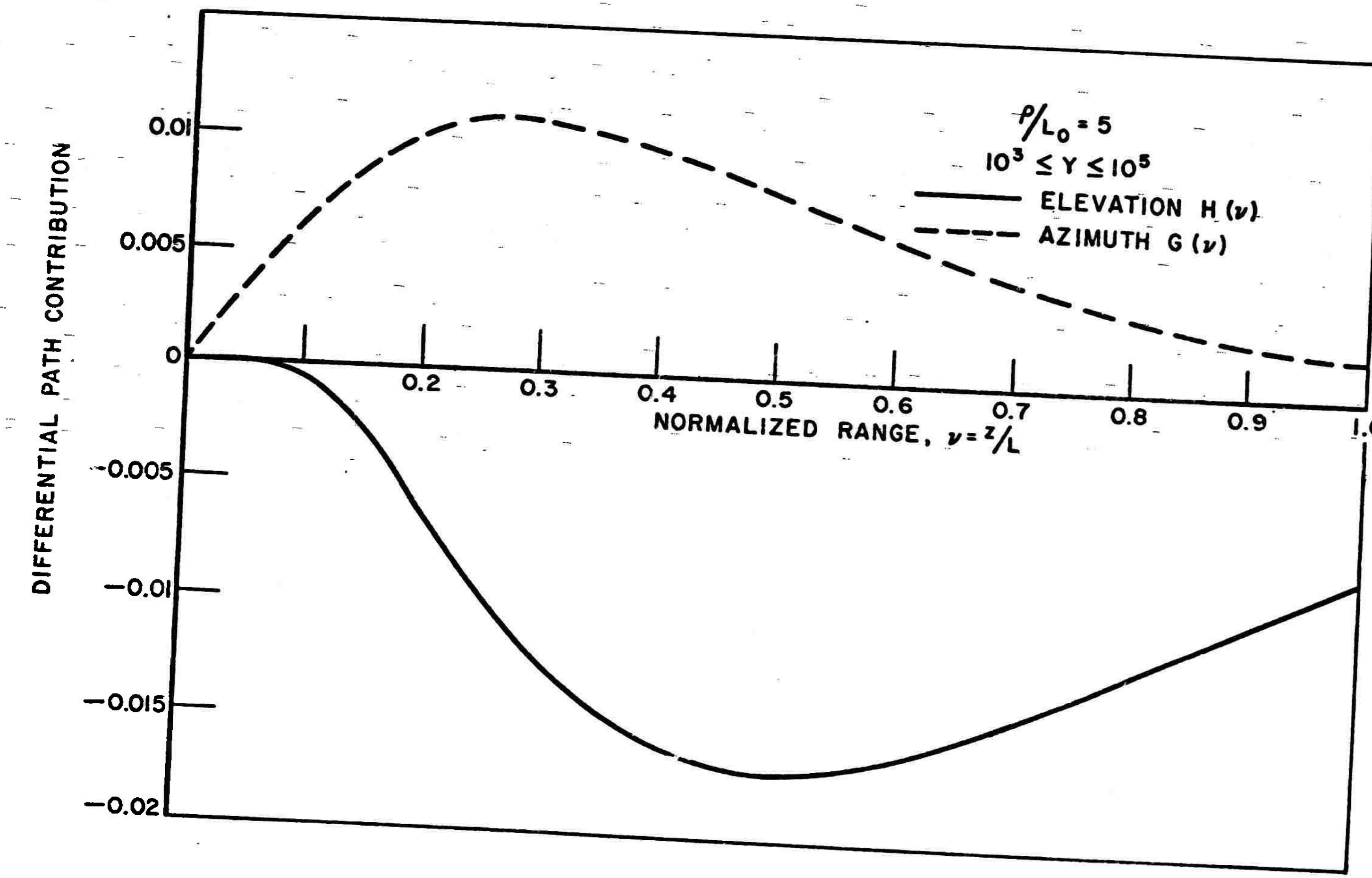


Fig. 7(a,b). Differential normalized angle of arrival correlation function contribution vs normalized range.



$\rho/L_0 = 5$
$10^3 \leq Y \leq 10^5$
ELEVATION H (ν)
AZIMUTH G (ν)
DIFFERENTIAL PATH CONTRIBUTION
NORMALIZED RANGE, $\nu = z/L$
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Fig. 7b. (Cont.)

BIBLIOGRAPHY OF OPTICAL PROPAGATION
IN A TURBULENT ATMOSPHERE

The final item is a continuation of the bibliography included in Ohio State University Report RF 2880-2, and includes journal articles dealing with the effects of a turbulent atmosphere on electromagnetic wave (generally light beam) propagation. These articles are listed by year and then alphabetically by author. One of the major features of this revised bibliography is the inclusion of Russian articles in Izv. Vuz. Radio Fizika that have not been translated into the English edition Soviet Radio Physics. These original Russian articles have English titles and abstracts.

There was also some additional information on authors and journals compiled from the complete bibliography. This information is presented in two additional listings.

List 1 tabulates the journals surveyed, the years and the number of entries found for the complete bibliography.

List 2 gives an alphabetical tabulation of the first authors in the complete bibliography and the years that they published.

1967

BOURKOV V G YASHIN YU IZV VUZ RADIOFIZ 10 1631-1638 1967
TO THE THEORY OF ELECTROMAGNETIC WAVE REFRACTION IN TWO-DIMENSIONAL INHOMOGENEOUS ISOTROPIC MEDIUM

GRACHEVA M E RADIOPHYS 10 424 1967 (10 775 1967)
INVESTIGATION OF THE STATISTICAL PROPERTIES OF STRONG FLUCTUATIONS IN THE INTENSITY OF LIGHT PROPAGATED THROUGH THE ATMOSPHERE NEAR THE EARTH

KAZANTSEV A N LUKIN D RAD ENG AND ELEC 12 1891-1910 1967
STUDY OF IONOSPHERE RADIOWAVE PROPAGATION

KOMISSAROV V M RADIOPHYS 10 270 1967 (10 498 1967)
FIELD OF A POINT SOURCE IN A RANDOMLY NONUNIFORM STRATIFIED MEDIUM

STREZH P E RAD ENG AND ELEC 12 1370-1377 1967
PLANE WAVES INCIDENT ON THE DIVISION BOUNDRY (VACUUM-INHOMOGENEOUS ISOTROPIC MEDIUM)

TATARSKI V I IZV VUZ RADIOFIZ 10 1762-1765 1967
ESTIMATION OF LIGHT DEPOLARIZATION BY TURBULENT ATMOSPHERIC INHOMOGENETIES

VVEDENSKIY B .. ET AL RAD ENG AND ELEC 12 1867-1890 1967
STUDY OF METER, DECIMETER, CENTIMETER, AND SUBMILLIMETER RADIOWAVE PROPAGATION

1968

ARSAEV I E KIMBER B E IZV VUZ RADIOFIZ 11 1377-1387 1968
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BAKHAREVA M F RAD ENG AND ELEC 13 983-988 1968
FREQUENCY SPACE CORRELATION OF THE FIELD FLUCTUATIONS, AMPLITUDE AND INTENSITY IN A MEDIUM WITH RANDOM INHOMOGENEITIES

BAKHAREVA M F RAD ENG AND ELEC 13 445-454 1968
FREQUENCY SPACE CORRELATION OF AMPLITUDE AND PHASE FLUCTUATIONS IN A MEDIUM WITH FORTUITOUS HETEROGENEITIES

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EQUATIONS FOR CORRELATION FUNCTIONS OF A WAVE BEAM IN A RANDOMLY-INHOMOGENEOUS MEDIUM

GURVICH A S ET AL IZV VUZ RADIOFIZ 11 66-71 1968
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GURVICH A S ET AL IZV VUZ RADIOFIZ 11 1360-1370 1968
FLUCTUATIONS OF THE PARAMETERS OF A LASER LIGHT WAVE PROPAGATING IN THE ATMOSPHERE

IZ''..OV A O RAD ENG AND ELEC 13 1155-1160 1968
FLUCTUATIONS OF THE AMPLITUDE AND PHASE OF PLANE MONOCHROMATIC WAVE OF SUBMILLIMETER WAVE AT PROPAGATION IN NEAR EARTH MEDIUM OF TURBULENT ATMOSPHERE WITH DUE REGARD FOR THE ABSORPTION IN WATER VAPOR

SHABELNIKOV A V RAD ENG AND ELEC 13 2115-2121 1968
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SHISHOV V I IZV VUZ RADIOFIZ 11 866-875 1968
THE THEORY OF WAVE PROPAGATION IN RANDOM MEDIA

ZAITSEV YU A ET AL IZV VUZ RADIOFIZ 11 1802-1811 1968
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ARTEMYEV A V RAD ENG AND ELEC 14 54 -546 1969
COHERENCE DISTORTION BY ATMOSPHERE
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CORRELATION OF FLUCTUATIONS OF AMPLITUDE AND PHASE OF PLANE MONOCHROMATIC WAVE OF SUBMILLIMETER BAND AT PROPAGATION IN TURBULENT ATMOSPHERE NEAR THE EARTH LAYER
KLYATSKIN V I IZV VUZ RADIOFIZ 12 723-726 1969
ON DISPERSION OF THE ANGLE OF ARRIVAL OF A PLANE LIGHT WAVE PROPAGATING IN A MEDIUM WITH RANDOM WEAK INHOMOGENEITIES
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FUNCTIONAL DESCRIPTION OF THE CHARACTERISTICS OF A PLANE LIGHT WAVE PROPAGATING IN A MEDIUM WITH RANDOM INHOMOGENEITIES OF THE REFRACTIVE INDEX
KLYATSKIN V I JETP 30 520-523 1970 (57 959-965 1969)
APPLICABILITY OF A MARKOV RANDOM PROCESS IN PROBLEMS RELATING TO THE PROPAGATION OF LIGHT IN A MEDIUM WITH RANDOM INHOMOGENEITIES
KON A I TATARSKI V I IZV VUZ RADIOFIZ 12 173-180 1969
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KRAVTSOV YU A ET AL IZV VUZ RADIOFIZ 12 674-685 1969
COMPLEX GEOMETRICAL OPTICS OF INHOMOGENEOUS ANISOTROPIC MEDIA
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PERMITIN G V FRAIMAN A IZV VUZ RADIOFIZ 12 1836-1841 1969
THE FIELD STRUCTURE NEAR CAUSTIC IN A MEDIUM WITH FLUCTUATIONS OF THE DIELECTRIC PERMITTIVITY
STOTSKIY A A RAD ENG AND ELEC 14 1547-1551 1969
MEASUREMENT OF FLUCTUATIONS OF PHASE DIFFERENCES OF CENTIMETER RADIOWAVES PROPAGATING IN THE ATMOSPHERE LAYER CLOSE TO THE EARTH
YZUMOV A O RAD ENG AND ELEC 14 1865-1867 1969
FREQUENCY SPECTRUM OF THE FLUCTUATIONS OF THE AMPLITUDE OF THE SUBMILLIMETER WAVEBAND PLANE MONOCHROMATIC WAVE PROPAGATING IN THE TURBULENT ATMOSPHERE NEAR THE EARTH

1970

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LIST 2

FIRST AUTHOR LIST

SUMMARY

In this interim report three areas of interst have been considered: averaging times for random data, phase structure function data and a continuation of a bibliography. The averaging time study reviewed approaches contained in the literature showing that the averaging time depends on the quantity of interest and on the particular criterion. The phase structure function computations gave curves intended to be useful in experiment design and data interpretation. The bibliography presented a listing of recent journal articles and author and journal lists.

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